From Spins to Circuits: An Introduction to Quantum Many-Body Physics

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Dr. Sunil Kumar Mishra

Department of Physics
Indian Institute of Technology (BHU) Varanasi

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Why Quantum Many-Body Physics?

- Systems of many interacting quantum particles are inherently close to the workings of nature.
- Many interacting quantum particles may lead to collective phenomena and interesting physics and technology.
- Examples: Superconductivity, magnetism, topological phases, quantum computers

Why Spin Systems are a good choice of quantum many-body system?

- A key to understanding quantum magnetism and magnetic materials.
- Spins (especially spin-1/2) are the simplest nontrivial quantum systems, with a finite-dimensional Hilbert space.
- Form a Minimal model to capture complex quantum phenomena, involving many-particle interactions.
- Interacting spin systems can exhibit a vast range of phenomena: quantum phase transitions, entanglement, topological order, etc.
- Important in the perspective of quantum computation.

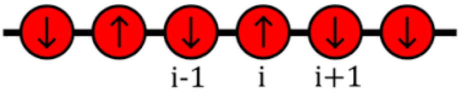


Figure: A simple example of spins in a lattice

Basic Motivations

- Spin models are an inherently quantum with no classical analogue
- Key systems to explore quantum phenomena like Superposition, Uncertainty & Entanglement.

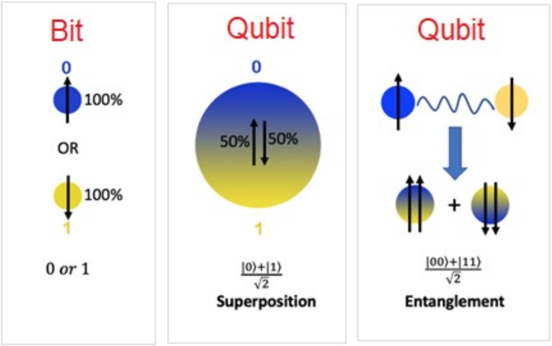


Figure: Quantum effects in terms of spins

Outline

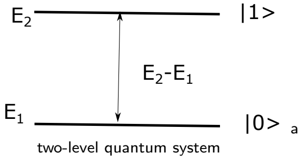
Important concepts in Quantum Mechanics

Important concepts in Quantum Mechanics

- Indeterminism $\rightarrow$ Indeterministic because the probability is inherent in quantum mechanics
- Interference $\rightarrow$ Even one quantum particle displays an interference pattern
- Uncertainty $\rightarrow$ Impossible to know both the quantum particle's position and momentum simultaneously and precisely.
- Superposition $\rightarrow$ A quantum particle can be in a linear combination state of allowable states.
- Entanglement $\rightarrow$ Correlations in some quantum states are stronger than any classical correlations

The simplest quantum system: Two-level system

- Simplest quantum system
  $\mathcal{H} = E_1|0\rangle\langle 0| + E_2|1\rangle\langle 1|$
  Two levels $|0\rangle$ and $|1\rangle$ with energies E_1 and E_2, respectively.
  Any quantum state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ (Superposition state),
  Probabilities α and β complex numbers such that $|\alpha|^2 + |\beta|^2 = 1$ (Indeterminism)
- If measured by some operator A, state $|\psi\rangle$ collapses to one of the eigenstates of A such that $A|\psi\rangle = c_a|a\rangle$, $c_a = < a|\psi\rangle$ (Measurement destroys the superposition state)
- quantum states evolve unitarily such that $|\psi'\rangle = U|\psi\rangle$ and $|\psi\rangle = U^\dagger|\psi'\rangle$, which implies $UU^\dagger = 1$ (Reversibility)



Spins: A Primer

The Spin 1/2 Space: A two-level quantum system

- Intrinsic angular momentum of quantum particles.
- Spin systems: two-level quantum states (qubits).
- As size of Hilbert space is 2, associated operators are of order 2×2.

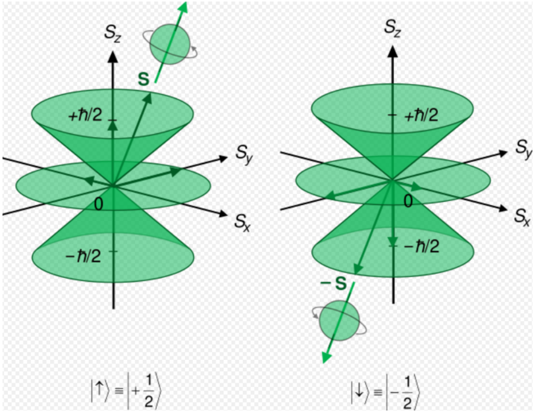
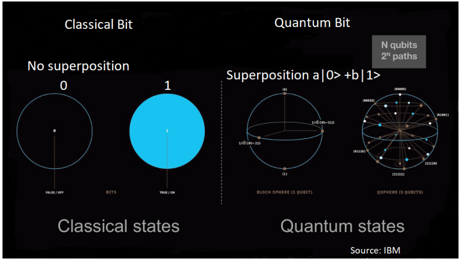
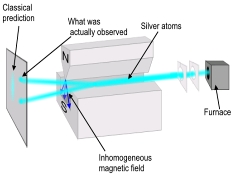


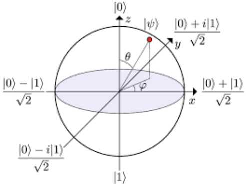
Figure: A representation of spins in the z basis

The Spin 1/2 State:Qubit



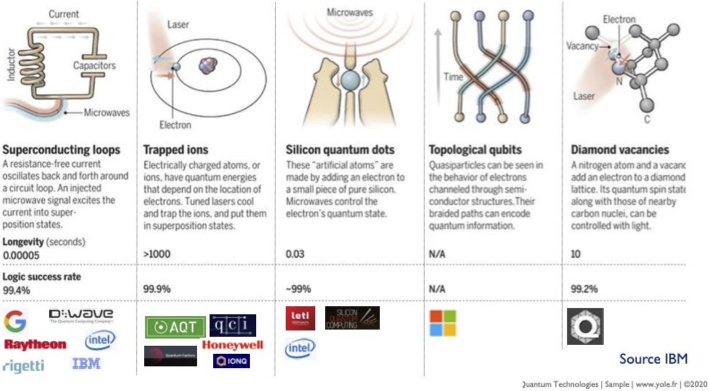


Stern-Gerlach experiment (1922), Figure Source: wikipedia



- Classical bits $\rightarrow$ 0(or ‘off’) and 1 (or ‘on’).
- $|0$ and $|1\rangle$ quantum equivalent of ‘0 ’ and ‘1’ classical bits and called as Quantum-bits or Qbits
- In general, a qubit state is a linear combination: $|\psi = \alpha|0\rangle + \beta|1\rangle, \; |\alpha|^2 + |\beta|^2 = 1$

Qubits in QC industry



Pauli Matrices and Spin Operators

Spin-1/2 operators defined by Pauli matrices:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Some useful properties:

- Commutation: $[\sigma_i, \sigma_j] = 2i\epsilon_{ijk}\sigma_k$.
- Anticommutation: $\{\sigma_i, \sigma_j\} = 2\delta_{ij}\,\mathbb{I}$

$\implies \sigma_i \cdot \sigma_j = \delta_{ij}\,\mathbb{I} + i\epsilon_{ijk}\sigma_k$

The corresponding spin operators are given as: S_x, S_y & S_z.

$$S_x = \frac{\hbar}{2}\sigma_x, \quad S_y = \frac{\hbar}{2}\sigma_y, \quad S_z = \frac{\hbar}{2}\sigma_z,$$

Role of Hamiltonian: Energy

Taking $\hat{z}$ as the conventional magnetic field direction, the energy of a dipole is given as $E = -\vec{\mu} \cdot \vec{B}_z$

- The magnetic moment is given as:

$$\vec{\mu} = -\frac{g_s \vec{S}}{\hbar} \mu_B$$

  which is dependent on the spin vector $\vec{S} = (S_x, S_y, S_z)$.
- Thus, the above energy is expressed by the operator $\hat{H} = -\tilde{\mu}_z S_z$.

Since the eigenvalues of the Hamiltonian give the possible values of energies, we have:

$$E_+ = \frac{-\gamma B_0 \hbar}{2}$$
$$E_- = \frac{-\gamma B_0 \hbar}{2}$$

$\tilde{\mu}/B_0$ gives the gyromagnetic ratio γ.

Role of Hamiltonian: Dynamics

The Hamiltonian also generates the dynamics within the system following the Schrodinger Equation.

- let χ_+ and χ_- be the eigenstates corresponding to $E_\pm$.
- $\implies \chi(t) = c_1(t)\chi_+ + c_2(t)\chi_-$

$$c_1(t) = a_0 e^{iE_+ t/\hbar}, \;\; c_2(t) = b_0 e^{iE_- t/\hbar}$$

$$|a_0|^2 + |b_0|^2 = 1, \text{(by Normalization)}$$

$\implies a_0 = sin(\alpha/2), b_0 = cos(\alpha/2)$, for a parameter α. This dynamics now describes the Larmour Precession.

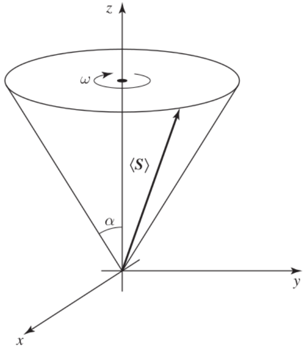


Figure: Larmour precession wrt $\langle S \rangle$, with $\omega = \gamma B_0$

An example: Light-Matter Interactions

The spin Hamiltonian can be used to model a simple example of light-matter interaction, describing the Rabi Oscillations.

Here, $\vec{B} = B_0 cos(\Omega_R t)\hat{x} + B_0 sin(\Omega_R t)\hat{y}$ from the incident $\mathcal{EM}$ field, with angular frequency Ω_R.

$$H = -\vec{\mu} \cdot \vec{B}$$

$$H = \omega' \Big(cos(\Omega_R t) S_x + sin(\Omega_R t) S_y \Big)$$

where $\omega' = \gamma B_0$

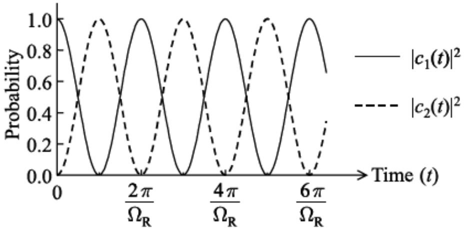


Figure: Oscillating occupation probabilities constituting a Rabi cycle

The Many-Body Hilbert Space

So far, we have a single spin Hamiltonian, interacting with an external field. Taking several spins introduces spin-spin interactions, while expanding the Hilbert space. Interactions imply variable-separation is not valid.

$$\mathcal{H}^{(N)} = \mathcal{H}_1 \otimes \mathcal{H}_2 ... \mathcal{H}_N = \otimes_{i=1}^{N} \mathcal{H}_i$$

Eg: For $N = 1$,
Basis $= \{|0\rangle_z, |1\rangle_z\}$
The basis vectors are given as:

$$\text{basis} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Eg: For $N = 2$,
Basis $= \{|00\rangle_z, |01\rangle_z, |10\rangle_z, |11\rangle_z\}$
The basis vectors are given as:

$$\text{basis} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

A Simple Many-Body Hamiltonian

Consider a $1D$ lattice with points labeled $\{1, 2, ..., i, i+1, ...N\}$.
Now, we may denote a spin operator S_i^α as the operator S_α, $(\alpha = x, y, z)$ acting on the lattice site i.
$\implies S_i^\alpha = \mathbb{I}_1 \otimes \mathbb{I}_2 \otimes ...S^\alpha... \otimes \mathbb{I}_N$. This further leads to: $[S_j^\alpha, S_k^\beta] = i\delta_{jk}\epsilon_{\alpha\beta\gamma}S_k^\gamma$

Now consider the example:

$$H = -J \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j$$

This Hamiltonian describes the spin-spin interaction between nearest-neighbor spins on our $1D$ lattice. This is the Heisenberg Model.

Interactions

Magnetic Properties via State

- The nature of interactions tunes the properties of the system, and also affects the ground state of the system.
- The ground state of the system is free from any excitations, and hence reflect the properties of the system at large.
- The adjoining figure shows the ground state corresponding to the observed macroscopic magnetic behavior.

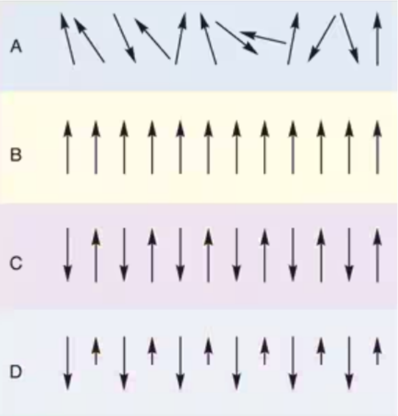


Figure: The ground states corresponding to paramagnetic, ferromagnetic, antiferromagnetic and ferrimagnetic systems

Origin of Interaction Strength

Pauli Exclusion Principle:

- Electrons are fermions; no two can occupy the same quantum state simultaneously.
- The total two-electron wavefunction must be antisymmetric:

$$\Psi_{\text{tot}}(1,2) = -\Psi_{\text{tot}}(2,1)$$

- For parallel spins (symmetric spin part), the spatial wavefunction must be antisymmetric to preserve overall antisymmetry.
- Antisymmetric spatial wavefunctions reduce overlap between electrons.

Reduced Coulomb Repulsion:

- Coulomb energy:

$$U \propto \left\langle \frac{1}{r_{12}} \right\rangle$$

- Greater overlap $\Rightarrow$ larger repulsion.
- Antisymmetric spatial wavefunctions $\Rightarrow$ reduced overlap $\Rightarrow$ lower Coulomb energy.
- This energy difference gives rise to the **exchange interaction**.
- $J > 0$ implies ferromagnetic and $J < 0$ implies antiferromagnetic ordering.

Ferromagnetic and Antiferromagnetic Ordering

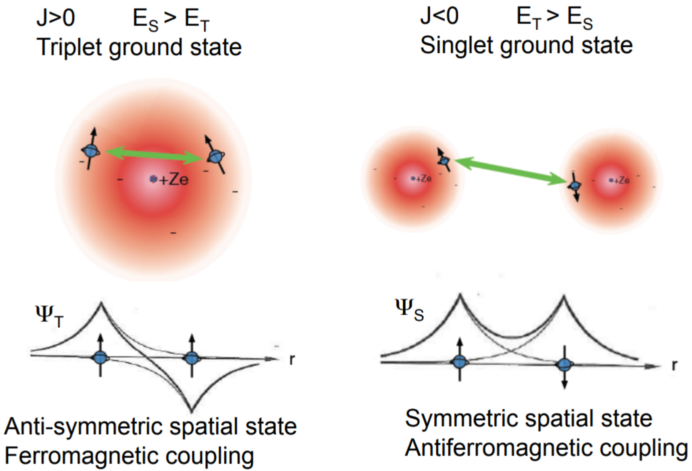


Figure: Interplay between Pauli exclusion and Coulomb repulsion leads to magnetic ordering

Direct and Indirect Exchange

- We have discussed about the nature of magnetism considering short range (direct) exchange.
- However, magnetism also has long range order and that is accomplished via indirect exchange, which requires the presence of a mediating element.

Examples for indirect exchange include:

1. Superexchange: Via nonmagnetic anion (e.g., O^{2-}), often AFM via Goodenough–Kanamori rules.
2. RKKY Interaction: In metals, mediated by conduction electrons; oscillates between FM/AFM with distance

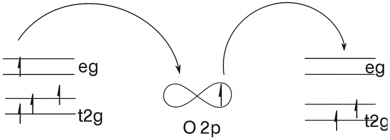


Figure: Indirect coupling via the superexchange mechanism.

Spin-Orbit Coupling

- More physical features are added by the spin-orbit coupling effect, given by the term $H_{soc} = \xi(r)\vec{L} \cdot \vec{S}$.
- SOC locks spin orientation to lattice directions (via **L**), breaking spin-rotational symmetry

H_{soc} leads to Dzyaloshinskii-Moriya interaction, between two spins S_1 and S_2 on a lattice bond r_{12} with no inversion center.

$$H_{DMI} = D_{ij}(H_{soc})\vec{S}_i \times \vec{S}_j$$

D_{ij} depends on H_{soc}, adds non-collinear spin texture resulting in net magnetization $M \neq 0$ in an otherwise collinear antiferromagnet (weak ferromagnetism).

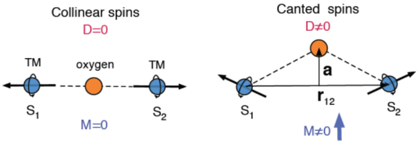


Figure: Non-collinearity in spins under H_{DMI}

Spin Configurations and Magnetic Ordering

- The FM and AFM order is reflected in ground the ground state at $T \approx 0K$.
- Upon increasing the temperature, thermal fluctuations appear, which destroy the magnetic order.
- This is phase transition, and the specific temperature is called as the Curie point T_C (for FM) and Neel point T_N (for AFM)

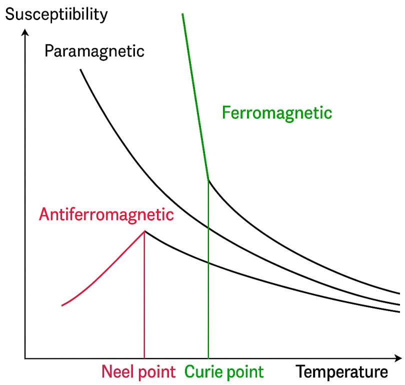


Figure: Phase diagram for FM/AFM to paramagnetic transition at T_C/T_N

Classical vs Quantum

Classical Phase Transitions

- Classical phase transitions are driven by a competition between magnetic ordering and thermal fluctuations.
- The simplest example is the transition between ferromagnetic and paramagnetic phases between the Ising model: $H = J\sum_i S_i^x S_{i+1}^x + h_x \sum_i S_i^x$.
- By the Landau paradigm the phase of a system corresponds to the free energy minima.

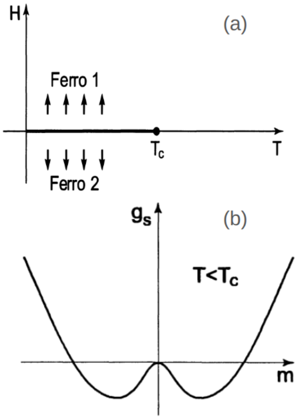


Figure: (a) Ferromagnetic and Paramagnetic states (b) along for the corresponding minima for the possible ferro states.

Quantum Versions of Phase Transition

- Contrary to classical phase transitions which require an element of thermal fluctuations,
- Quantum Phase Transitions occur at zero kelvin where thermal fluctuations are impossible.
- Instead such phase transitions occur due to quantum fluctuations.
- Since they occur at zero temperature limit, ground states serve as an efficient discriminator.

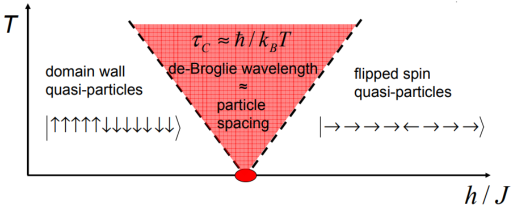


Figure: Phase structure with under quantum phase transition, along with contrasting ground states.

The above phase structure is for:

$$H = J\sum_i S_i^x S_{i+1}^x + h_x \sum_i S_i^x + h_z \sum_i S_i^z$$

Entanglement in Quantum Systems

Entanglement between subsystems A and B within the entire system defined by ψ is given as:

$$S_L(\psi) = \eta\left(1 - tr\rho_A^2\right) = \eta\left(1 - tr\rho_B^2\right)$$

η is defined such that $S_L(\psi) \in [0, 1]$

$$\eta = \frac{1}{d(d+1)}, d = min(d_A, d_B)$$

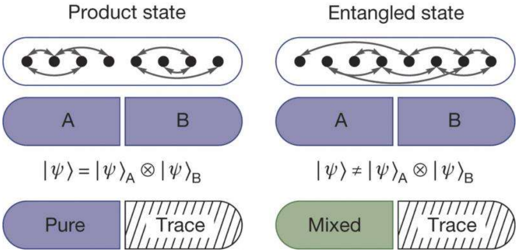


Figure: Pure and mixed partial states [a]

[a] Choi et.al. Nature **528** 77-83(2015)

Entanglement as a probe

- Entanglement within the ground state serves as a metric for classifying the phase of the system
- The dependence on entanglement with system size follows the relation:

$$S(A/\bar{A}) = \alpha \partial A - \gamma$$

where ∂A represents the boundary of the system, and γ is a constant called as topological quantum entropy

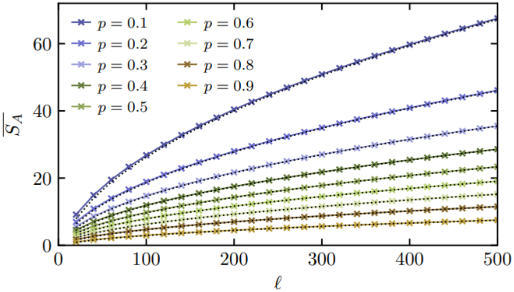


Figure: An example of the entanglement being used as a probe for quantum phase transition induced by measurements

Ref: Claeys *at.al* Phys. Rev. Research **4**, 043212 (2022)

Various spin models

1d spin models

$$\mathcal{H} = -g\mu_B h \sum_{j=1}^{L} S_j^z - \sum_{j=1}^{L}(J_x S_j^x S_{j+1}^x + J_y S_j^y S_{j+1}^y + J_z S_j^z S_{j+1}^z).$$

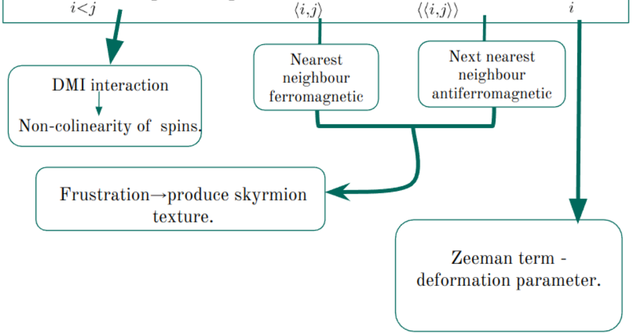
$h =$magnetic field, J_x, J_y, J_z are exchange interaction coefficients, L spins

- $J_x = 0, J_y = 0, J_z \neq 0 \rightarrow$ Longitudinal Ising model (Exact solution, Ising 1925)
- $J_x = J_y = J_z = J \rightarrow$ XXX Heisenberg model (Exact solution, Bethe 1931)
- $J_x \neq J_y, J_z = 0 \rightarrow$ XY model (Exact solution, Lieb, Shulz, Mattis 1961)
- $J_x \neq 0, J_y, J_z = 0 \rightarrow$ Transverse Ising model (Exact solution, similar to XY model, Lieb, Shulz, Mattis 1961)
- $J_x = J_y \neq J_z \rightarrow$ XXZ model (Exact solution, Orbach 1958)
- $J_x \neq J_y \neq J_z \neq 0 \rightarrow$ XYZ model (Exact solution, Baxter 1972)

2d spin model: Quantum Skyrmions

Hamiltonian

$$\hat{H} = \sum_{i<j} \mathbf{D}_{i,j} \left[\hat{S}_i \times \hat{S}_j\right] + J_1 \sum_{\langle i,j \rangle} \hat{S}_i \hat{S}_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} \hat{S}_i \hat{S}_j + B \sum_i \hat{S}_i^z$$



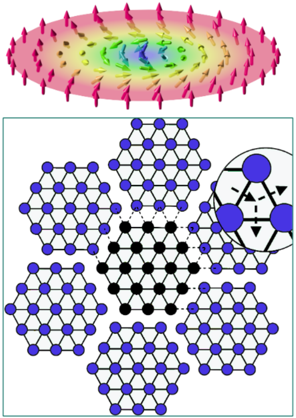


Figure: Lattice structure for given hamiltonian and skyrmion texture

Ref: Vipin V. *et.al.*, Phys. Rev. B **107**, L100419 (2023)

VBS Solids

- Valence bond solid (VBS) states are quantum ground states with short-range singlet pairing between neighboring spins.
- VBS order breaks lattice translational symmetry, leading to dimerized or more complex bond patterns.
- The AKLT model is a well-known example where the exact VBS ground state can be written analytically.

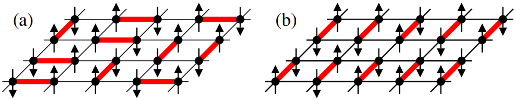


Figure: An example of the VBS order in (*a*) Resonating valence band spin liquid (*b*) columnar VBS.

Ref: Sandvik, AIP Conf. Proc. **1297**, 135 (2010)

Model for Quantum Phase Transition

VBS States also show Quantum Phase Transition

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + J' \sum_{\langle\langle ij \rangle\rangle_{dimer}} \vec{S}_i \cdot \vec{S}_j$$

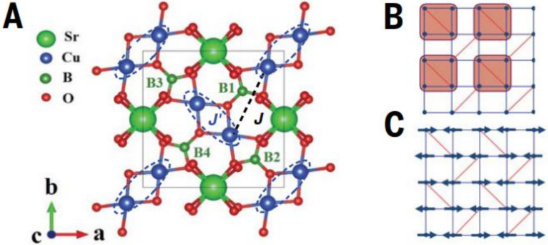


Fig. **A.** Atomic structure of the $SrCu_2(BO_3)_2$ plane. Pairs of Cu form spin dimers. Each unit cell contains 4 B ions
B. The PS phase in the square lattice with J and J' bonds.
C. The AFM phase that breaks the O(3) symmetry

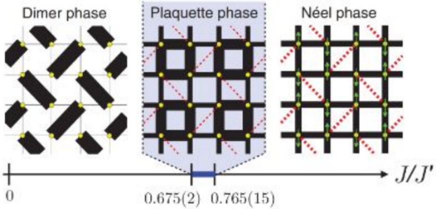


Fig. The phase diagram of the Shastry-Sutherland model. The arrows in the right panel illustrate the Neel order. In between the well-established dimer and Neel phase we find a phase with plaquette long-range order

Ref: Cui *et al.* Proximate deconfined quantum critical point in SrCu2(BO3)2, Science **380** ,1179-1184(2023)

Field Description and the Role of Numerics

Mapping Spins to Hard-Core Bosons

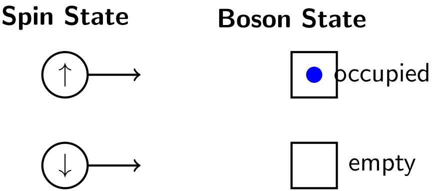
- Spin-$\frac{1}{2}$ operators, $S_i^+ = S_i^x - iS_i^y$

$$S_i^+ = |\uparrow\rangle\langle\downarrow|, \quad S_i^- = |\downarrow\rangle\langle\uparrow|, \quad S_i^z = \frac{1}{2}(|\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow|)$$

- Mapped to hard-core boson operators:

$$S_i^+ \leftrightarrow b_i^\dagger, \quad S_i^- \leftrightarrow b_i, \quad S_i^z = b_i^\dagger b_i - \frac{1}{2}$$

- Hard-core constraint: $(b_i^\dagger)^2 = 0$ — no double occupancy.
- Useful for reformulating spin models in bosonic language.



Jordan-Wigner Transformation: Hard Bosons to Fermions

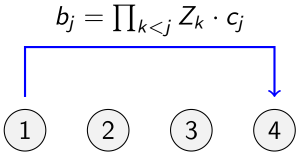
- Hard-core bosons obey:

$$\{b_i, b_j^\dagger\} = \delta_{ij}, \quad (b_i^\dagger)^2 = 0$$

- However, they commute on different sites: $[b_i, b_j] = 0$, unlike fermions.
- Jordan-Wigner transformation maps hard bosons to fermions:

$$b_j = \left(\prod_{k<j} Z_k\right) c_j, \quad b_j^\dagger = \left(\prod_{k<j} Z_k\right) c_j^\dagger$$

  where $Z_k = 1 - 2c_k^\dagger c_k$
- The string ensures correct anticommutation: $\{c_i, c_j^\dagger\} = \delta_{ij}$



Bogoliubov Transformation: Fermions to Quasiparticles

- Consider a quadratic fermionic Hamiltonian:

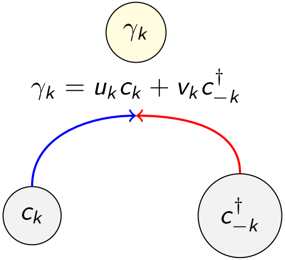
$$H = \sum_k \left[\epsilon_k c_k^\dagger c_k + \Delta_k c_k^\dagger c_{-k}^\dagger + \Delta_k^* c_{-k} c_k \right]$$

- This describes **pairing** between fermions at opposite momenta.
- Bogoliubov transformation defines new fermionic quasiparticles:

$$\gamma_k = u_k c_k + v_k c_{-k}^\dagger, \quad \gamma_{-k} = u_k c_{-k} - v_k c_k^\dagger$$

- The coefficients u_k, v_k are chosen to **diagonalize** the Hamiltonian:

$$H = \sum_k E_k \left(\gamma_k^\dagger \gamma_k - \frac{1}{2} \right)$$



Mbeng *et.al.* SciPost Phys. Lect. Notes **82** (2024)

Non-integrable Models: Computational Methods

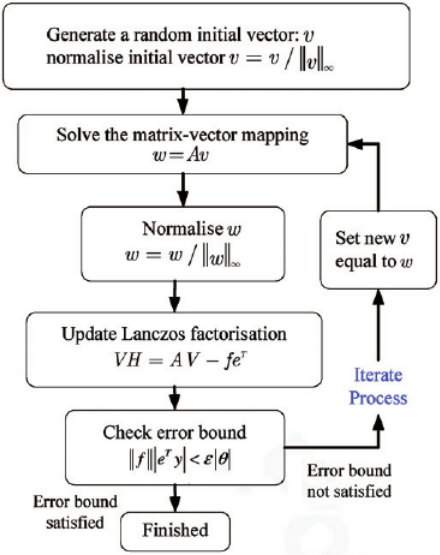
- Non-integrable models lack an extensive set of conserved quantities.
- Cannot be mapped to independent single-particle modes.
- Many-body interactions dominate dynamics and eigenstates.
- Require full many-body Hilbert space treatment.
- Exhibit quantum chaos, thermalization, and eigenstate complexity.

Hence, computational methods are required, but they have **exponentially scaling** computational requirements.

Number of sites	Number of states	Hamiltonian size in memory
4	16	2048 B
9	512	2 MB
16	65536	34 GB
25	33554432	9 PB
36	6.872e10	40 ZB

Iterative Diagonalization: Lanczos Method

- Iterative method for finding extremal eigenvalues of large sparse matrices.
- Builds a tridiagonal matrix in a Krylov subspace.
- Efficient for computing ground states in many-body quantum systems.

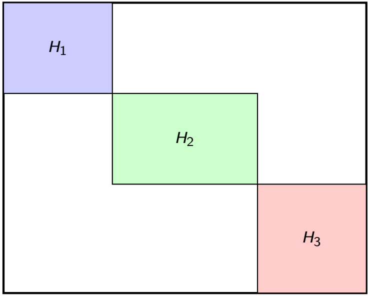


Block-Diagonal Hamiltonians and Symmetry

- Time evolution of an observable:

$$\frac{d}{dt}\langle \hat{O}(t)\rangle = \frac{i}{\hbar}\langle [H, \hat{O}]\rangle$$

- If $[H, \hat{O}] = 0$, then $\langle \hat{O}(t)\rangle$ is conserved over time.
- Such symmetries imply the Hilbert space splits into invariant subspaces.
- The Hamiltonian becomes block-diagonal in the basis of simultaneous eigenstates of commuting operators, greatly reducing computational cost.



Off-diagonal = 0

Ref: Jung *et.al.* Journal of the Korean Physical Society, **Volume 76**, pages 670–683, (2020)

Quantum Circuits

Quantum Circuits

- Quantum circuits manipulate qubit states via unitary gates.
- A single-qubit gate is represented by a 2×2 unitary matrix.
- Any single-qubit unitary gate corresponds to a rotation on the Bloch sphere.
- General rotation:

$$U(\vec{n}, \theta) = e^{-i\frac{\theta}{2}\vec{n}\cdot\vec{\sigma}}$$

  where $\vec{n}$ is the rotation axis, θ the angle, and $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$.
- Circuit depth and structure determine the entanglement and computational complexity.

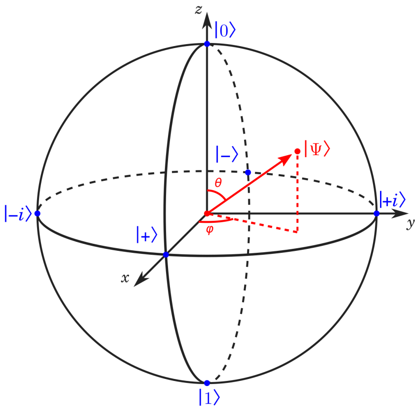


Figure: The block sphere representation of qubits. Quantum gates are rotation operations on the bloch sphere.

Heisenberg Evolution as Circuit

- Trotterization approximates time evolution by sequentially applying local terms: $e^{-iHt} \approx \prod e^{-ih_{ij}\Delta t}$.
- Each local term h_{ij} (e.g., $\vec{S}_i \cdot \vec{S}_j$) is implemented using rotation and entangling gates.

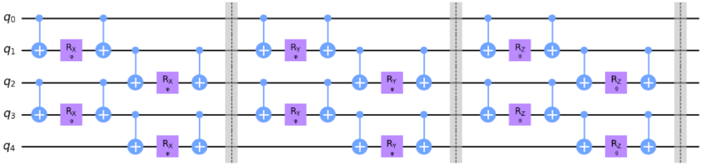


Figure: The circuit for $N = 5$, and $t = 3$

Implementation Results

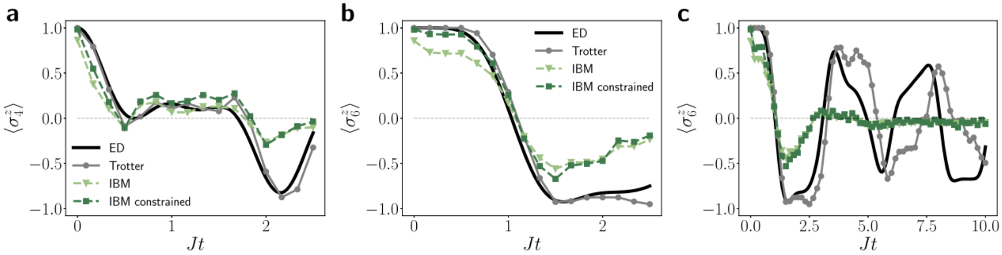


Figure: Results for a global quench for TFIM from a domain wall initial state. (a, b) The local magnetization of the fourth and sixth spins of the chain with $N = 6$, and (c) is for the sixth spin for longer times

Ref: Smith *et.al.* npj Quantum Information **Volume 5**, Article number: 106 (2019)

Dynamical Aspects

Quantum Chaotic Models

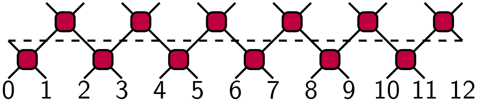
Setting: Dynamics generated by a brickwall circuit made up by repeating units of $U \in \mathcal{DU}(q)$, for $q = 2, 3$.

Motivation: A class of circuit which allows for maximally chaotic behaviour in a minimal model.

We define the circuit for a single time step as (with PBC):

$$\mathcal{U} = \left(\otimes_{i \in Z_{even}} U^{i,i+1} . \otimes_{j \in Z_{odd}} U^{j,j+1} \right).$$

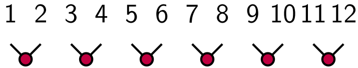
This maybe graphically denoted as:

$\mathcal{U} =$ 

.

The remaining parameter is the intial state, denoted as:

$$|\psi\rangle_{initial} = \frac{1}{q^{N/2}} \left(\sum_{i,j=0}^{q-1} m_{ij} |ij\rangle \right)^{\otimes N} ,$$

$|\psi_{init}\rangle =$ 

Random Quantum Circuits

- Random quantum circuits consist of layers of randomly chosen local unitary gates.
- They break symmetries and avoid conservation laws, mimicking non-integrable dynamics.
- Such circuits exhibit fast entanglement growth and thermalization-like behavior.
- Useful for studying quantum chaos, scrambling, and eigenstate thermalization hypothesis (ETH).

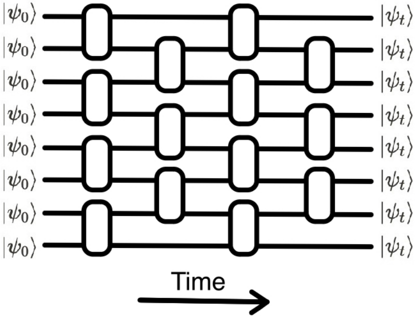


Figure: A schematic random quantum circuit, with each gate corresponding to randomly selected parameters

Conclusions

Recap

- **Spins: A Primer** — Spin-1/2 systems as minimal models for quantum two-level systems (qubits).
- **Interactions** — Magnetic properties arise from spin-spin interactions, exchange mechanisms, and symmetry considerations.
- **Classical vs Quantum** — Contrasting phase transitions, with quantum fluctuations dominating at $T = 0$.
- **Some Examples** — Realizations like Floquet models, skyrmions, and VBS illustrate emergent phenomena.
- **Field Description and Numerics** — Reformulating spin models via mappings enables computational approaches.
- **Quantum Circuits** — Time evolution and dynamics encoded using unitary gate-based circuits.
- **Dynamical Aspects** — Random and chaotic circuits as minimal models for entanglement spreading and ETH.

Summary

- Spin systems offer a foundational framework to study collective quantum phenomena.
- Many-body models reveal rich behavior — from magnetism and phase transitions to entanglement and chaos.
- Quantum circuits bridge the gap between theoretical models and physical implementations in quantum computing.
- Analytical tools and numerical methods together help explore non-integrable dynamics beyond solvable regimes.
- The study of quantum dynamics and entanglement is central to modern developments in condensed matter and quantum information science.